Mathematics Areas of Focus: Grade 3

Mission: *Through mathematics, students communicate, make connections, reason, and represent the world quantitatively in order to pose and solve problems.*

Standard 4.1 Number and Numerical Operations

All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways.

Big Idea: Numeric reasoning involves fluency and facility with numbers.

4.1.3 A. Number Sense

Descriptive Statement: *Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.*

Essential Questions	Enduring Understandings
▪ How do mathematical ideas interconnect and build on one another to produce a coherent whole? (4.5C1; 4.5C6)** ▪ How can we compare and contrast numbers? (4.5A4)** ▪ How can counting, measuring, or labeling help to make sense of the world around us?	▪ One representation may sometimes be more helpful than another; and, used together, multiple representations give a fuller understanding of a problem. ▪ A quantity can be represented numerically in various ways. Problem solving depends upon choosing wise ways. ▪ Numeric fluency includes both the understanding of and the ability to appropriately use numbers.

Areas of Focus	Comments and Examples
1. **Use real-life experiences, physical materials, and technology to construct meanings for numbers (unless otherwise noted, all indicators for grade 3 pertain to these sets of numbers as well).** ▪ **Whole numbers through hundred thousands** ▪ **Commonly used fractions (denominators of 2, 3, 4, 5, 6, 8, 10) as part of a whole, as a subset of a set, and as a location on a number line**	**Instructional/Assessment Focus:** • It is important to note that the sets of numbers specified in this CPI also apply to the other grade 3 mathematics CPIs, including, for example, 4.1.3A5 and 4.1.3B6.
2. **Demonstrate an understanding of whole number place value concepts.**	**Sample Assessment Items:** • **Multiple Choice (MC):** What is the value of the 3 in 75,314? a. thirty * b. three hundred c. three thousand d. thirty thousand • **MC:** Using the digits 1 - 5 only once, what is the largest even number you can make with a 5 in the hundreds place? a. 54,321 b. 54,312 * c. 43,512 d. 32,514
3. **Identify whether any whole number is odd or even.**	**Suggested Instructional/Assessment Strategies:** • Students read literature that incorporates basic number concepts in an enjoyable and engaging way (e.g., *Even Steven and Odd Todd*, a Hello Reader by Kathryn Cristaldi et al. Scholastic, Inc., 1996).
4. Explore the extension of the place value system to decimals through hundredths.	**Instructional/Assessment Focus:** • This content should be introduced at this grade level, but mastery of the content is not assessed in statewide assessment at this grade level.
5. **Understand the various uses of numbers.** ▪ **Counting, measuring, labeling (e.g., numbers on baseball uniforms)**	**Instructional/Assessment Focus:** • Refers not only to whole through hundred thousands, but also commonly used fractions (denominators of 2, 3, 4, 5, 6, 8, 10), as specified in 4.1.3A1.
6. **Compare and order numbers.**	**Instructional/Assessment Focus:** • Refers not only to whole through hundred thousands, but also commonly used fractions (denominators of 2, 3, 4, 5, 6, 8, 10), as specified in 4.1.3A1.

4.1.3 B. Numerical Operations

Descriptive Statement: *Numerical Operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.*

Essential Questions	Enduring Understandings
▪ What makes a computational strategy both effective and efficient? (4.5D1)**	▪ Computational fluency includes understanding not only the meaning, but also the appropriate use of numerical operations.
▪ How do operations affect numbers?	▪ The magnitude of numbers affects the outcome of operations on them.
▪ How do mathematical representations reflect the needs of society across cultures? (An essential question with broad applicability across multiple standards) (4.5C5)**	▪ In many cases, there are multiple algorithms for finding a mathematical solution, and those algorithms are frequently associated with different cultures.

Areas of Focus	Comments and Examples
1. Develop the meanings of the four basic arithmetic operations by modeling and discussing a large variety of problems. ▪ **Addition and subtraction: joining, separating, comparing** ▪ **Multiplication: repeated addition, area/array** ▪ **Division: repeated subtraction, sharing**	**Instructional/Assessment Focus:** • The focus in grade 3 is on developing meanings for multiplication and division. Students should have developed meanings for addition and subtraction in grades 1 and 2.
2. Develop proficiency with basic multiplication and division number facts using a variety of fact strategies (such as "skip counting" and "repeated subtraction").	**Sample Assessment Item:** • **Short Constructed Response (SCR):** Brett is taking care of his neighbor's dog for 7 days. Brett needs to let the dog outside 3 times a day. In all, how many times will Brett let the dog out? *(This item would appear on a non-calculator portion of the statewide assessment. Answer: 21 times or 21)*
3. Construct, use, and explain procedures for performing whole number calculations with: ▪ **Pencil-and-paper** ▪ **Mental math** ▪ **Calculator**	**Sample Assessment Items:** • **MC:** 376 + 119 + 85 = a. 460 * b. 580 c. 1,230 d. 1,345 *(This item would appear on a non-calculator portion of the statewide assessment.)* • **SCR:** Mark has a stamp collection. He has 22 stamps from Japan, 34 from Canada, and 17 from Mexico. How many stamps does he have in all? (*This item would appear on a non-calculator portion of the statewide assessment. Answer: 73 stamps.*)
4. Use efficient and accurate pencil-and-paper procedures for computation with whole numbers. ▪ **Addition of 3-digit numbers** ▪ **Subtraction of 3-digit numbers** ▪ **Multiplication of 2-digit numbers by 1-digit numbers**	**Sample Assessment Items:** • **MC:** Find the exact answer: 110 marbles + 70 marbles = a. 18 marbles b. 81 marbles * c. 180 marbles d. 810 marbles *(This item would appear on a non-calculator portion of the statewide assessment.)* • **MC:** Find the exact value of 24 x 7 a. 31 b. 141 c. 148 * d. 168 *(This item would appear on a non-calculator portion of the statewide assessment.)* • **SCR:** Find the exact answer: 110 + 70 = _____ *(This item would appear on a non-calculator portion of the statewide assessment. Answer: 180)* • **SCR:** Find the exact answer: 145 + 281 + 62 = _____ *(This item would appear on a non-calculator portion of the statewide assessment. Answer: 488)* • **SCR:** John had 365 pennies. He gave 56 pennies to his sister. How many pennies does John have left? _____ *(This item would appear on a non-calculator portion of the statewide assessment. Answer: 309 pennies or 309¢ or $3.09)*
5. Count and perform simple computations with money. ▪ **Cents notation (¢)**	**Sample Assessment Items:** • **SCR:** What is the fewest number of coins needed to make 99¢, using only pennies and dimes? *(Answer: 18)* • **MC:** What is the fewest number of coins needed to make 99¢, using only pennies and dimes? a. 8 b. 14 * c. 18 d. 19

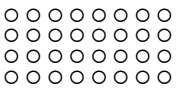
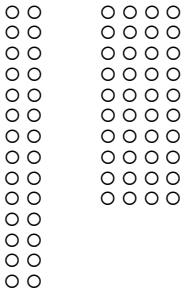
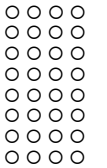
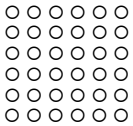
	• **Extended Constructed Response (ECR):** A juice machine charges 65¢ for a can of juice and accepts only nickels, dimes, and quarters. The machine requires exact change.
	  • Show a combination of the exact number of coins you could put in the juice machine to get a can of juice.
	  • Is there another combination of coins you could use to get a can of juice? Show your work or explain your answer.

6. Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.	
7. Check the reasonableness of results of computations.	**Suggested Instructional/Assessment Strategy:** • Note the connection to Estimation CPI 4.1.3C4.

4.1.3 C. Estimation

Descriptive Statement: *Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.*

Essential Questions	Enduring Understandings
▪ How can we decide when to use an exact answer and when to use an estimate?	▪ Context is critical when using estimation.

Areas of Focus	Comments and Examples
1. **Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set**.	**Sample Assessment Item:** • **MC:** Pat has a coin collection displayed this way:  Which of the following displays contains fewer coins than Pat's? * a.  b. c.  d. 
2. Construct and use a variety of estimation strategies (e.g., rounding and mental math) for estimating both quantities and the result of computations.	**Instructional/Assessment Focus:** ▪ Assessment of this CPI is frequently within the context of CPI 4.1.3C4 or one or more of the other content CPIs. ▪ A common student mistake is rounding single-digit numbers (*e.g., mentally converting 36 x 7 to 40 x 10 = 400, rather than to 40 x 7 = 280).* **Sample Assessment Items:** ▪ **MC:** Estimate 123 + 685. The sum is between which numbers? a. 400 and 600 *b. 700 and 900 c. 1,000 and 1,200 d. 1,300 and 1,500 *(This item would appear on a non-calculator portion of the statewide assessment.)* ▪ **MC:** Find the exact answer: 900 – 201 = * a. 699 b. 700 c. 701 d. 799 *(This item would appear on a non-calculator portion of the statewide assessment.)* ▪ **MC:** Estimate 423 - 174. The difference is between which numbers? a. 0 and 199 * b. 200 and 399 c. 400 and 599 d. 600 and 799 *(This item would appear on a non-calculator portion of the statewide assessment.)* ▪ **MC:** Sandra traveled 458 miles to North Carolina, then 231 miles from North Carolina to West Virginia, and finally 340 miles home. Which of the following best describes the distance Sandra traveled? a. 600 mi b. 800 mi * c. 1000 mi d. 1200 mi

3. Recognize when an estimate is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.	**Instructional/Assessment Focus:** • Assessment of this CPI and demonstration of this understanding is frequently within the context of one or more of the other content CPIs. • Student articulation of this understanding is expected to be evolving in grade 3. Statewide assessment of the concept should receive greater attention in later grades.
4. **Use estimation to determine whether the result of a computation (either by calculator or by hand) is reasonable.**	**Sample Assessment Items:** ▪ **ECR:** Your friend Susan said that 454 + 42 = 432. Use estimation to explain why you think Susan is wrong. ▪ **ECR:** Sam and Kelly were adding the numbers of students in their two schools. Sam told Kelly that 367 + 417 = 600. Use estimation to explain if you think Sam is right or wrong and why. ▪ **ECR:** Peter discovered that the school enrollment this year is 150 less than last year, when there were 826 students. Kiesha told Peter that there are now about 575 students. Use estimation to explain why you think Kiesha is right or wrong.

Standard 4.2 Geometry and Measurement

All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze phenomena.

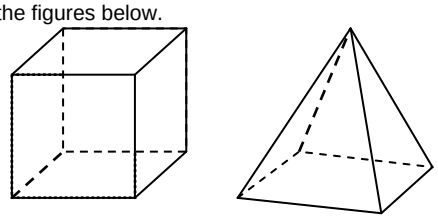
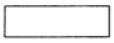
Big Idea *Geometry***:** Spatial sense and geometric relationships are a means to solve problems and make sense of a variety of phenomena.

Big Idea *Measurement***:** Measurement is a tool to quantify a variety of phenomena.

4.2.3 A. Geometric Properties

Descriptive Statement: *This includes identifying, describing and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.*

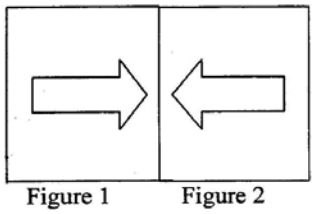
Essential Questions	Enduring Understandings
▪ How can spatial relationships be described by careful use of geometric language? ▪ How do geometric relationships help us to solve problems and/or make sense of phenomena?	▪ Geometric properties can be used to construct geometric figures. (4.5D1; 4.5D2; 4.5E3)** ▪ Geometric relationships provide a means to make sense of a variety of phenomena.

Areas of Focus	Comments and Examples
1. Identify and describe spatial relationships of two or more objects in space. ▪ **Direction, orientation, and perspectives (e.g., which object is on your left when you are standing here?)** ▪ **Relative shapes and sizes**	
2. Use properties of standard three-dimensional and two-dimensional shapes to identify, classify, and describe them. ▪ **Vertex, edge, face, side, angle** ▪ **3D figures – cube, rectangular prism, sphere, cone, cylinder, and pyramid** ▪ **2D figures – square, rectangle, circle, triangle, pentagon, hexagon, octagon**	**Sample Assessment Item:** • **ECR:** Look at the figures below.  • Name each figure. • How many faces does each figure have? • Explain how you counted the faces.
3. Identify and describe relationships among two-dimensional shapes. ▪ **Same size, same shape** ▪ **Lines of symmetry**	**Sample Assessment Items:** • **MC:** Which figure appears to be the same size and same shape as Shape S?  Shape S a.  b.  c.  * d.  • **MC:** Which of these letters has a line of symmetry? a. **P** b. **F** * c. **T** d. **L**
4. Understand and apply concepts involving lines, angles, and circles. ▪ **Line, line segment, endpoint**	
5. Recognize, describe, extend, and create space-filling patterns.	

4.2.3 B. Transforming Shapes

Descriptive Statement: *This includes identifying, describing and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.*

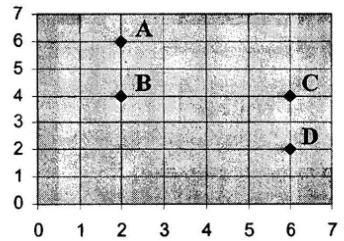
Essential Questions	Enduring Understandings
▪ What situations can be analyzed using transformations and symmetries? (4.5E1; 4.5E2; 4.5E3)**	▪ Shape and area can be conserved during mathematical transformations.

Areas of Focus	Comments and Examples
1. Describe and use geometric transformations (slide, flip, turn).	**Sample Assessment Item:** • **MC:** Which of the following describes the change in Figure 1 to Figure 2?  Figure 1 Figure 2 a. slide b. turn right * c. flip d. turn left
2. Investigate the occurrence of geometry in nature and art.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI. Assessment of this CPI is generally within the context of one or more of the other content CPIs.

4.2.3 C. Coordinate Geometry

Descriptive Statement: *Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.*

Essential Questions	Enduring Understandings
▪ How can we best represent and verify geometric/algebraic relationships? (4.5C2; 4.5D2; 4.5E1; 4.5E2; 4.5F5)**	▪ Reasoning and/or proof can be used to verify or refute conjectures or theorems in geometry (4.5D1; 4.5D3; 4.5D4; 4.5D5; 4.5F5)** ▪ Coordinate geometry can be used to represent and verify geometric/algebraic relationships.

Areas of Focus	Comments and Examples
1. Locate and name points in the first quadrant on a coordinate grid.	**Sample Assessment Item:** • **MC:** Which point is located at (2,6) ?  * a. A b. B c. C d. D

4.2.3 D. Units Of Measurement

Descriptive Statement: *Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.*

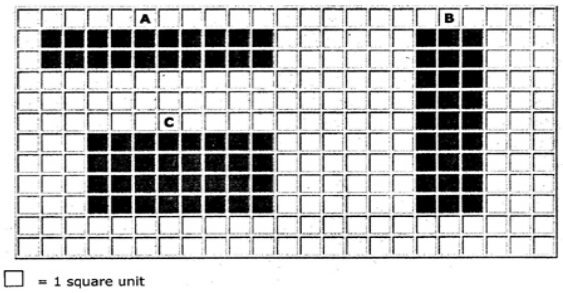
Essential Questions	Enduring Understandings
▪ How can measurements be used to solve problems? (4.5A6)**	▪ Everyday objects have a variety of attributes, each of which can be measured in many ways. ▪ What we measure affects how we measure it. (4.5A4; 4.5A6)** ▪ Measurements can be used to describe, compare, and make sense of phenomena.

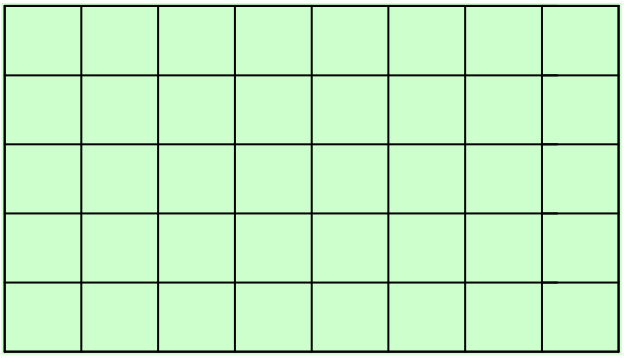
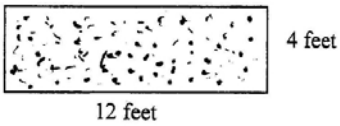
Areas of Focus	Comments and Examples
1. Understand that everyday objects have a variety of attributes, each of which can be measured in many ways.	
2. Select and use appropriate standard units of measure and measurement tools to solve real-life problems. **▪ Length – fractions of an inch (1/4, 1/2), mile, decimeter, kilometer** **▪ Area – square inch, square centimeter** **▪ Weight – ounce** **▪ Capacity – fluid ounce, cup, gallon, milliliter**	**Instructional/Assessment Focus:** • Both instruction and assessment will focus on students actually measuring, rather than on reading or hearing about others measuring. Students are expected to have become familiar with both inches and centimeters in second grade. **Sample Assessment Items:** • **MC:** Use your ruler to answer this question. To the nearest centimeter, what is the length of line segment AB? **A B** [*Line segment provided for student would have an actual length of approximately 3 centimeters.*] a. 34 cm b. 33 cm c. 4 cm * d. 3 cm • **MC:** Which is the best unit to describe the amount of water in a swimming pool? a. cup b. pint c. quart * d. gallon
3. Incorporate estimation in measurement activities (e.g., estimate before measuring).	

4.2.3 E. Measuring Geometric Objects

Descriptive Statement: *This area focuses on applying the knowledge and understandings of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.*

Essential Questions	Enduring Understandings
▪ How can measurements be used to solve problems? (4.5A6)**	▪ Everyday objects have a variety of attributes, each of which can be measured in many ways. ▪ What we measure affects how we measure it. (4.5A4; 4.5A6)** ▪ Measurements can be used to describe, compare, and make sense of phenomena.

Areas of Focus	Comments and Examples
1. Determine the area of simple two-dimensional shapes on a square grid.	**Sample Assessment Items:** • **ECR:** Carefully examine each of the three rectangles shown below.  • Find the area of each rectangle. • Determine which rectangle has the greatest area. • Show your work and clearly explain your answer.

	• **MC:** Amanda wants to cover the top of her doll's table with colored paper. The top of the table is shown below.  *[Grid provided for student would be made up of squares one centimeter on each side.]* How many square centimeters of paper does Amanda need if each square equals 1 square centimeter? a. 5 b. 8 c. 26 * d. 40
2. Determine the perimeter of simple shapes by measuring all of the sides.	**Instructional/Assessment Focus:** • Both instruction and assessment will focus on students actually measuring, rather than on reading or hearing about others measuring. **Sample Assessment Item:** • **ECR:** Rob is building a fence around his rectangular garden as shown below.  How many feet of fencing does Rob need? a. 16 feet b. 20 feet * c. 32 feet d. 48 feet
3. Measure and compare the volume of three–dimensional objects using materials such as rice or cubes.	**Instructional/Assessment Focus:** • Students are expected to solve problems (4.5A2)** involving this recognition. • The emphasis in grade 3 would be on the "measure," rather than the "compare."

Standard 4.3 Patterns and Algebra

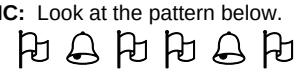
All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes.

Big Idea: Algebra provides language through which we communicate the patterns in mathematics.

4.3.3 A. Patterns

Descriptive Statement: *Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.*

Essential Questions	Enduring Understandings
▪ How can change be best represented mathematically? (4.5C1; 4.5F1; 4.5F2; 4.5F3; 4.5F4)** ▪ How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? (4.5C1)**	▪ The symbolic language of algebra is used to communicate and generalize the patterns in mathematics. ▪ Algebraic representation can be used to generalize patterns and relationships.

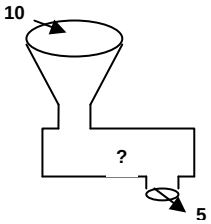
Areas of Focus	Comments and Examples
1. Recognize, describe, extend, and create patterns. ▪ **Descriptions using words and number sentences/expressions** ▪ **Whole number patterns that grow or shrink as a result of repeatedly adding, subtracting, multiplying by, or dividing by a fixed number (e.g., 5, 8, 11, . . . or 800, 400, 200, . . .)**	**Sample Assessment Items:** • **MC:** Look at the pattern below.  Which letter pattern matches the shape pattern? a. F B F B F B b. F B D F B D c. F F F B B B * d. F B F F B F

	• **MC:** If this pattern continues, what is the next number? 4, 7, 10, 13, 16, 19, … a. 21 * b. 22 c. 23 d. 24 • **MC:** If this pattern continues, what is the next number? 1, 2, 4, 8, … a. 10 b. 12 c. 14 * d. 16 • **MC:** Look at the pattern below:  If the pattern continues, how many hearts will be in the next figure? a. 5 b. 10 * c. 15 d. 20

4.3.3 B. Functions & Relationships

Descriptive Statement: *The function concept is one of the most fundamental unifying ideas of modern mathematics. Student begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations which express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.*

Essential Questions	Enduring Understandings
▪ How are patterns of change related to the behavior of functions? (4.5F1; 4.5F2; 4.5F3; 4.5F4)**	▪ Patterns and relationships can be represented graphically, numerically, symbolically, or verbally. (4.5E1)**

Areas of Focus	Comments and Examples
1. Use concrete and pictorial models to explore the basic concept of a function. ▪ **Input/output tables, T-charts**	**Sample Assessment Items:** • **MC:** Which rule is applied to the input numbers in order to get the output numbers? **Input** / **Output**: 3 → 6; 5 → 8; 6 → 9; 8 → 11 * a. Add 3 b. Subtract 3 c. Multiply by 2 d. Divide by 2 • **MC:** When 10 is dropped into this machine, it comes out as 5.  When 16 is dropped in, it comes our as 8. When 4 is dropped in, it comes our as 2. If 8 is dropped into the machine, what number will it come out as? a. 3 * b. 4 c. 5 d. 6

4.3.3 C. Modeling

Descriptive Statement: *Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).*

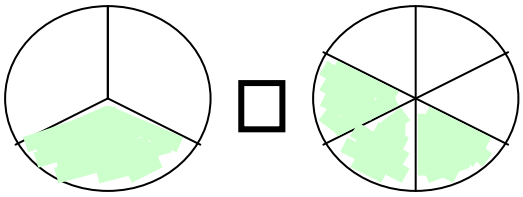
Essential Questions	Enduring Understandings
▪ How can we use mathematical models to describe physical relationships? (4.5E2)**	▪ Mathematical models can be used to describe and quantify physical relationships. (4.5E2)**
▪ How can we use physical models to clarify mathematical relationships? (4.5E3)**	▪ Physical models can be used to clarify mathematical relationships. (4.5E3)**

Areas of Focus	Comments and Examples
1. Recognize and describe change in quantities. ▪ **Graphs representing change over time (e.g., temperature, height)**	
2. Construct and solve simple open sentences involving addition or subtraction (e.g., 3 + 6 = __, n = 15 – 3, 3 + __ = 3, 16 – c = 7).	**Sample Assessment Items:** • **MC:** Kamala bought a box of crayons for 29¢. She also bought a coloring book for 65¢. Which number sentence shows how much money Kamala spent on the crayons and coloring book? a. 65¢ - 29¢ = ___ b. ___ + 29¢ = 65¢ * c. 29¢ + 65¢ = ___ d. 65¢ - ___ = 29¢ • **MC:** What does the *p* equal in $3 + p = 15$? a. 3 b. 5 * c. 12 d. 18

4.3.3 D. Procedures

Descriptive Statement: *Techniques for manipulating algebraic expressions - procedures - remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.*

Essential Questions	Enduring Understandings
▪ What makes an algebraic algorithm both effective and efficient? (4.5D1)**	▪ Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole. ▪ Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra. (4.5D1; 4.5D3; 4.5D4; 4.5D5)**

Areas of Focus	Comments and Examples
1. Understand and apply the properties of operations and numbers. ▪ **Commutative (e.g., 3 x 7 = 7 x 3)** ▪ **Identity element for multiplication is 1 (e.g., 1 x 8 = 8)** ▪ **Any number multiplied by zero is zero**	**Instructional/Assessment Focus:** • While recognizing that 3x7 and 7x3 yield the same answer, grade 3 students would not necessarily be expected to label that as the commutative property.
2. Understand and use the concepts of equals, less than, and greater than to describe relations between numbers. ▪ **Symbols (= , < , >)**	**Sample Assessment Items:** • **MC:** Jake is 47 inches tall. Mike is 39 inches tall. Which of the following correctly compares the height of each child? a. 39 > 47 b. 39 = 47 c. 47 < 39 * d. 47 > 39 • **MC:** Compare the shaded regions. Which symbol belongs in the square?  * a. < b. > c. = d. None of the above

Standard 4.4 Data Analysis, Probability, and Discrete Mathematics

All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics, and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data.

Big Idea *Data Analysis*: Reading, understanding, interpreting, and communicating data are critical in modeling a variety of real-world situations, drawing appropriate inferences, making informed decisions, and justifying those decisions.

Big Idea *Probability*: Probability quantifies the likelihood that something will happen and enables us to make predictions and informed decisions.

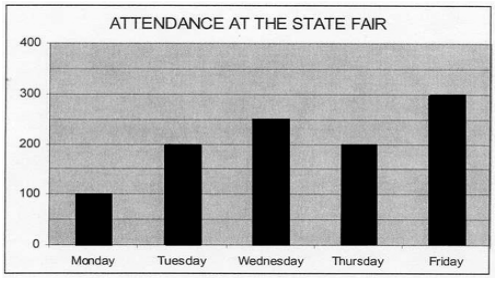
Big Idea *Discrete Mathematics*: Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.

4.4.3 A. Data Analysis

Descriptive Statement: *In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.*

Essential Questions	Enduring Understandings
▪ How can the collection, organization, interpretation, and display of data be used to answer questions? (4.5A4; 4.5A6; 4.5E1; 4.5E2; 4.5F1; 4.5F6)**	▪ The message conveyed by the data depends on how the data is collected, represented, and summarized. (4.5A6; 4.5D6; 4.5E1; 4.5E2; 4.5E3)** ▪ The results of a statistical investigation can be used to support or refute an argument. (4.5D1; 4.5D3; 4.5D5; 4.5E2; 4.5E3; 4.5F6)**

Areas of Focus	Comments and Examples
1. Collect, generate, organize, and display data in response to questions, claims, or curiosity. ▪ Data collected from the classroom environment	**Instructional/Assessment Focus:** • The actual collection of data would be more a part of classroom instruction or performance assessment, rather than a part of statewide assessment. • Assessment of this CPI is frequently within the context of CPI 4.4.3A2.
2. Read, interpret, construct, analyze, generate questions about, and draw inferences from displays of data. ▪ **Pictograph, bar graph, table**	**Instructional/Assessment Focus:** • Students are expected to have become familiar with tally charts in second grade. **Sample Assessment Items:** • **MC:**



The graph above shows the number of tickets sold for the first five days of the week. How many tickets were sold on the third day of the week?

a. 100 b. 150 c. 200 * d. 250

• **MC:** Sue is having some friends over for pizza. She surveyed what toppings they would like on their pizza.

Toppings	Number of Votes
Cheese	|||
Pepperoni	𝍸 |
Sausage	||||
Mushrooms	
Onions	|

What can Sue most likely conclude from her survey?

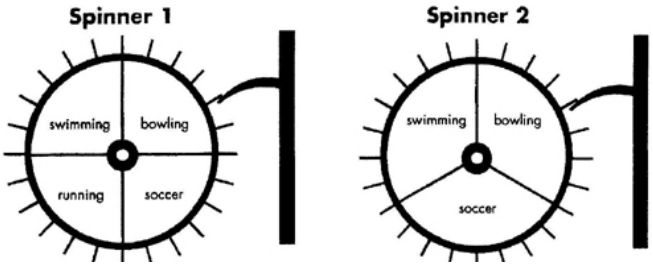
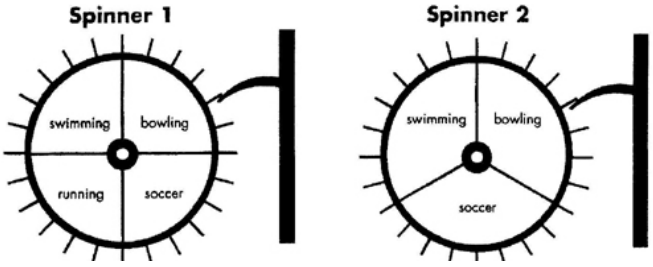
a. More of Sue's friends like cheese than pepperoni pizza.
* b. Sausage is the group's second-favorite type pizza.
c. A pizza with both pepperoni and mushrooms should be ordered.

	d. Sue needs to order only one onion pizza. • **ECR:** Construct a bar graph to accurately display the following data. **Number of books read by fourth-grade students** **Student** | Jan | Kyra | Laramie | Mickey **Number of Books** | 7 | 3 | 8 | 2 ▪ Carefully label the graph and give it an appropriate title. (*This item would include a blank grid on the statewide assessment.)*

4.4.3 B. Probability

Descriptive Statement: *Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.*

Essential Questions	Enduring Understandings
▪ How can experimental and theoretical probabilities be used to make predictions or draw conclusions? (4.5D5; 4.5D6)**	▪ Experimental results tend to approach theoretical probabilities after a large number of trials.

Areas of Focus	Comments and Examples
1. Use everyday events and chance devices, such as dice, coins, and unevenly divided spinners, to explore concepts of probability. ▪ Likely, unlikely, certain, impossible ▪ More likely, less likely, equally likely	**Instructional/Assessment Focus:** • The exploration using dice, coins, and unevenly divided spinners is largely instructional, and generally assessed indirectly on statewide assessments. • Familiarity with the concepts and vocabulary in the bullets is frequently assessed within the context of CPI 4.4.3B2. **Sample Assessment Items:** • **ECR:** It is Cheryl's turn to spin a spinner to choose an activity for her physical education class. Cheryl will spin one of the spinners below to decide what the class will do.  Cheryl's favorite activity is swimming. • Find the probability of landing on swimming using Spinner 1. • Find the probability of landing on swimming using Spinner 2. • Decide which spinner Cheryl should choose if she wants to go swimming. • Explain why Cheryl should choose this spinner. (*Note: Students may provide answers to the first two bullets without using fractions. Also, fractions, if used, would not need to be reduced at this grade level.*) • **MC:** It is Cheryl's turn to spin a spinner to choose an activity for her physical education class. Cheryl will spin one of the spinners below to decide what the class will do.  If Cheryl spins spinner 2, what activity is impossible to choose?

	a. swimming b. bowling * c. running d. soccer
2. Predict probabilities in a variety of situations (e.g., given the number of items of each color in a bag, what is the probability that an item picked will have a particular color). ▪ **What students think will happen (intuitive)** ▪ **Collect data and use that data to predict the probability (experimental)**	**Sample Assessment Item:** • **MC:** Orlando has a bag of 10 marbles that contains 4 red marbles and 6 blue marbles. If Orlando reached into the bag without looking and picked one marble, what is the probability that he would pick a blue marble? a. 1 out of 10 b. 4 out of 10 * c. 6 out of 10 d. 10 out of 10

4.4.3 C. Discrete Mathematics - Systematic Listing And Counting

Descriptive Statement: *Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.*

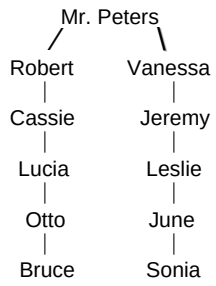
Essential Questions	Enduring Understandings
▪ How can attributes be used to classify data/objects? ▪ What is the best way to solve this? What counting strategy works best here?	▪ Grouping by attributes (classification) can be used to answer mathematical questions. (4.5E1; 4.5E3)** ▪ Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

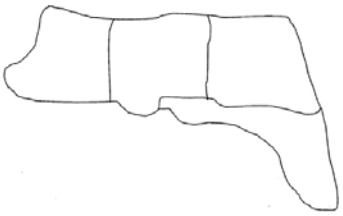
Areas of Focus	Comments and Examples
1. Represent and classify data according to attributes, such as shape or color, and relationships. ▪ **Venn diagrams** ▪ **Numerical and alphabetical order**	
2. Represent all possibilities for a simple counting situation in an organized way and draw conclusions from this representation. ▪ **Organized lists, charts**	**Sample Assessment Item:** **MC:** Roseanne has 3 sweatshirts: a grey one, a green one, and a red one. She also has 2 pairs of jeans: a blue pair and a black pair. If an outfit consists of one sweatshirt and one pair of jeans, how many different outfits can Roseanne make? a. 8 * b. 6 c. 5 d. 3

4.4.3 D. Discrete Mathematics - Vertex-Edge Graphs And Algorithms

Descriptive Statement: *Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms," and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.*

Essential Questions	Enduring Understandings
▪ How can visual tools such as networks (vertex-edge graphs) be used to answer questions? (4.5E1; 4.5E3)** ▪ How can algorithmic thinking be used to solve problems?	▪ Optimization is finding the best solution within given constraints. ▪ Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Areas of Focus	Comments and Examples
1. Follow, devise, and describe practical sets of directions (e.g., to add two 2-digit numbers).	**Sample Assessment Item:** • **MC:** The mathematics club uses this phone tree to remind members about club activities. Mr. Peters calls Robert and Vanessa, and then each student calls the person whose name is listed under their name. This continues until every student is called.  Which student will Leslie call?

	a. Jeremy * b. June c. Lucia d. Otto
2. Explore vertex-edge graphs. ▪ Vertex, edge ▪ Path	**Instructional Focus:** • This content should be introduced at this grade level, but mastery of the content is not assessed in statewide assessment at this grade level.
3. Find the smallest number of colors needed to color a map.	**Sample Assessment Item:** • **MC:** To color the following map, you want to use as few colors as possible. What is the fewest number of colors you can use so that no areas that touch are the same color?  a. 2 * b. 3 c. 4 d. 5

Standard 4.5 Mathematical Processes

All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.

While no additional big ideas, essential questions, or enduring understandings are listed for this standard, the mathematical processes are imbedded in the content-specific ideas, questions, and understandings delineated for the first four standards. References to the relevant processes can be found above.

4.5 A. Problem Solving

Descriptive Statement: *Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.*

Areas of Focus	Comments and Examples
1. Learn mathematics through problem solving, inquiry, and discovery.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4
2. Solve problems that arise in mathematics and in other contexts. ▪ **Open-ended problems** ▪ **Non-routine problems** ▪ **Problems with multiple solutions** ▪ **Problems that can be solved in several ways**	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
4. Pose problems of various types and levels of difficulty.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4
5. Monitor their progress and reflect on the process of their problem solving activity.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4. • Instructionally, this is more applicable to later grade levels.

4.5 B. Communication

Descriptive Statement: *Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in themselves as mathematics learners. It also enables teachers to better monitor student progress.*

Areas of Focus	Comments and Examples
1. Use communication to organize and clarify mathematical thinking. ▪ **Reading and writing**	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

▪ **Discussion, listening, and questioning**	
2. Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.	• Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Analyze and evaluate the mathematical thinking and strategies of others.	• Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
4. Use the language of mathematics to express mathematical ideas precisely.	• Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 C. Connections

Descriptive Statement: *Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.*

Areas of Focus	Comments and Examples
1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4. • Instructionally, this is more applicable to later grade levels.
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4. • Instructionally, this is more applicable to later grade levels
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4
4. Apply mathematics in practical situations and in other disciplines.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4

4.5 D. Reasoning

Descriptive Statement: *Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.*

Areas of Focus	Comments and Examples
1. Recognize that mathematical facts, procedures, and claims must be justified.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4
2. Use reasoning to support their mathematical conclusions and problem solutions.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Select and use various types of reasoning and methods of proof.	**Instructional/Assessment Focus:** • This may be included in classroom enrichment activities at this grade level, but is more of a focus at secondary grade levels.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Make and investigate mathematical conjectures. ▪ Counterexamples as a means of disproving conjectures ▪ Verifying conjectures using informal reasoning or proofs	**Instructional/Assessment Focus:** • This may be included in classroom enrichment activities at this grade level, but is more of a focus at higher grade levels.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.	**Instructional/Assessment Focus:** • This is more of a focus at secondary grade levels.

4.5 E. Representations

Descriptive Statement: *Representations refers to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.*

Areas of Focus	Comments and Examples
1. Create and use representations to organize, record, and communicate mathematical ideas. ▪ **Concrete representations (e.g., base-ten blocks or algebra tiles)** ▪ **Pictorial representations (e.g., diagrams, charts, or tables)** ▪ **Symbolic representations (e.g., a formula)** ▪ **Graphical representations (e.g., a line graph)**	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
2. Select, apply, and translate among mathematical representations to solve problems.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Use representations to model and interpret physical, social, and mathematical phenomena.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 F. Technology

Descriptive Statement: *Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and should also become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).*

Areas of Focus	Comments and Examples
1. Use technology to gather, analyze, and communicate mathematical information.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4. • This may be included in classroom enrichment activities at this grade level, but is more of a focus at higher grade levels.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4. • This may be included in classroom enrichment activities at this grade level, but is more of a focus at higher grade levels.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Use computer software to make and verify conjectures about geometric objects.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
6. Use computer-based laboratory technology for mathematical applications in the sciences.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.